1 NUMBER SYSTEMS

Introduction

- **Number:** It means that something you would count.
- **Natural numbers:** They are the counting numbers, denoted by N.
  $\therefore$ N = {1, 2, 3, 4, 5, ...}
- **Whole numbers:** The natural numbers together with zero are called whole numbers and denoted by W.
  $\therefore$ W = {0, 1, 2, 3, 4, 5, ...}
- **Negative numbers:** The numbers that are opposite to the positive numbers are called negative numbers.
- **Integers:** It is a whole number (not a fractional number) that can be positive, negative or zero and denoted by Z.
  $\therefore$ Z = {..... –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5 .....}
  Note: Numbers like $\frac{3}{4}, \frac{1}{2}$, 1.15, 6.7 etc. are not integers.
  (*i*) Negative integers = {..... –6, –5, –4, –3, –2, –1}
  (*ii*) Positive integers = {1, 2, 3, 4, 5, 6 .....}
  (*iii*) Non-negative integers = {0, 1, 2, 3, 4 .....}
- **Rational numbers:** A number that can be written in the form $\frac{p}{q}$ is called a rational number, where p and q are integers and $q \neq 0$. It is denoted by Q.
  For example: If $p = 4$ and $q = 3$, then $r = \frac{p}{q} = \frac{4}{3}$ is a rational number.

Irrational Numbers

- **Irrational numbers:** A number that cannot be written in the form $\frac{p}{q}$ is called an irrational number, where p and q are integers and $q \neq 0$.
  For example: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$, 0.161016100161000161.....
  Note: (*i*) Zero is not an irrational number. It is a rational number.
  (*ii*) Surds are the irrational numbers.
- **Real numbers:** The collection of all set of rational and irrational numbers together are known as real numbers, denoted by R.
  There is a unique real number corresponding to every point on the number line. Conversely, corresponding to each real number, there is a unique point on the number line. Hence, number line is called real number line.

Real Numbers and their Decimal Expansions

- **Decimal expansion of rational number:** The decimal expansion of a rational number is either terminating or non-terminating recurring.
  For example: (*i*) 0.44444 .... = $0.\overline{4}$ (*ii*) 1.323232 ..... = $1.\overline{32}$
  (*iii*) 0.3525252 ...... = $0.3\overline{52}$ etc.

- **Decimal expansion of irrational number:** The decimal expansion of an irrational number is non-terminating non-recurring.

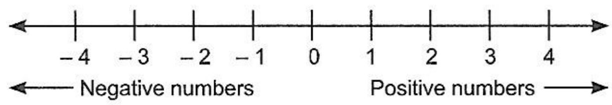
  For example: (*i*) $\sqrt{2} = 1.4142135623$ .... (*ii*) $\pi = 3.1415926535$ ....

  (*iii*) 1.202002000200002 .... (*iv*) 2.16016001600016000016 ....

Representing Real Numbers on the Number Line

Number line/Real line/Real Number line

- It is a horizontal line such that corresponding to every real number, there is a point on the real number line, and corresponding to every point on the number line, there exists a unique real number.
- Since zero is a real number, so corresponding to zero, there is a unique point on the number line called origin, and to the right of origin, all points are positive numbers, while left of this point, all points represent negative numbers.



- The point corresponds to real number with a terminating decimal expansion on the number line can be visualised by the process of sufficient successive magnification.

SOLVED QUESTIONS BASED ON EXERCISES 1.1, 1.2, 1.3 AND 1.4

Very Short Answer Type Questions [1 Mark]

1. Write a rational number between rational numbers $\frac{1}{9}$ and $\frac{2}{9}$. **[CBSE 2014]**

Sol. A rational number between $\frac{1}{9}$ and $\frac{2}{9}$ is $= \frac{\frac{1}{9}+\frac{2}{9}}{2} = \frac{3}{9\times 2} = \frac{1}{6}$

2. Write a rational number not lying between $-\frac{1}{5}$ and $-\frac{2}{5}$. **[CBSE 2013]**

Sol. $-\frac{3}{5}$

3. Write $\frac{p}{q}$ form of the number $0.\bar{3}$. **[CBSE 2011]**

Sol. $\frac{1}{3}$

4. Write $\frac{327}{500}$ in decimal form.

Sol. $\frac{327}{500} = 0.654$

5. Write a rational number which does not lie between the rational numbers $-\frac{2}{3}$ and $-\frac{1}{5}$. **[CBSE 2011]**

Sol. $\frac{3}{10}$

6. Write two irrational numbers.

Sol. $\sqrt{7}, \sqrt{11}, \sqrt{12}, \sqrt{14}$ etc. *(any two)*

Short Answer Type Questions I [2 Marks]

7. Express $3.\bar{2}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **[CBSE 2011]**

Sol. Let $x = 3.\bar{2} = 3.2222 \ldots$ ...(*i*)

Here, only one digit is repeating.

Multiplying both sides by 10, we get

$$10x = 32.222 = 32.\bar{2} \quad \ldots(ii)$$

Subtracting (*i*) from (*ii*), we get $10x - x = 32.\bar{2} - 3.\bar{2} = 29$

$$9x = 29$$

$$x = \frac{29}{9}$$

8. Express $18.\overline{48}$ in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **[CBSE 2011]**

Sol. Let $x = 18.\overline{48} = 18.484848 \ldots$ ...(*i*)

Here, two digits are repeating.

Multiplying both sides by 100, we get

$$100x = 1848.4848 \ldots = 1848.\overline{48} \quad \ldots(ii)$$

Subtracting (*i*) from (*ii*), we get

$$100x - x = 1848.\overline{48} - 18.\overline{48}$$

or $99x = 1830$

or $x = \frac{1830}{99} = \frac{610}{33}$

9. Express $\frac{4}{7}$ in decimal form and state the kind of decimal expansion. **[CBSE 2016]**

Sol. $\frac{4}{7} = 0.571428571428 \ldots = 0.\overline{571428}$

Therefore, the decimal expansion of the given rational number is non-terminating recurring (repeating).

10. Find the rational number of the form $\frac{p}{q}$ corresponding to the decimal representation 0.222 ..., where p and q are integers and $q \neq 0$. **[CBSE 2016]**

Sol. Let $x = 0.222 \ldots = 0.\bar{2}$

Here, only one digit is repeating.

Multiplying both sides by 10, we get

$$10x = 2.2222 \ldots = 2.\bar{2} = 2 + 0.\bar{2} = 2 + x$$

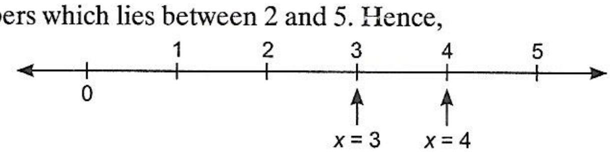
$\Rightarrow$ $10x - x = 2$

$\Rightarrow$ $9x = 2$

$\Rightarrow$ $x = \frac{2}{9}$

11. Represent the real numbers given by $2 < x < 5$ on the number line.

Sol. 3 and 4 are the real numbers which lies between 2 and 5. Hence,



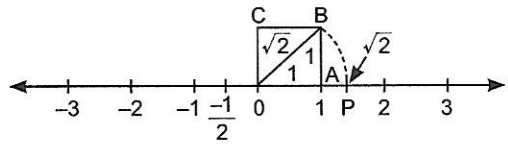
12. Represent $\sqrt{2}$ on the real number line. **[CBSE 2011]**

Sol. Using Pythagoras theorem, $\sqrt{2} = \sqrt{1^2 + 1^2}$

$\Rightarrow$ $OB = \sqrt{OA^2 + AB^2} = \sqrt{2}$

Hence, take OA = 1 unit on the number line and AB = 1 unit, which is perpendicular to OA.
With O as centre and OB as radius, we draw an arc to intersect the number line at P. Then P corresponds to $\sqrt{2}$ on the number line as shown in figure.

Clearly, $$OP = OB = \sqrt{2}$$



13. Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$. Given that $\frac{1}{7} = 0.\overline{142857}$.

Sol. Given $$\frac{1}{7} = 0.\overline{142857}$$

$\therefore$ $$\frac{2}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

One of the non-terminating non-recurring number between $\frac{1}{7}$ and $\frac{2}{7}$ is 0.150150015000150000015...

Short Answer Type Questions II [3 Marks]

14. Examine whether $\sqrt{2}$ is rational or irrational number. **[CBSE 2016]**

Sol. Let us find the square root of 2 by division method.

We get $\sqrt{2} = 1.41421356...$

Thus, the process will neither terminate nor a block of digits will repeat in the process. Hence, $\sqrt{2}$ has a non-terminating and non-recurring decimal expansion.

$\therefore$ $\sqrt{2}$ is an irrational number.

	1.41421356...
1	$2.\overline{00}\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}$
	1
24	100
	96
281	400
	281
2824	11900
	11296
28282	60400
	56564
282841	383600
	282841
2828423	10075900
	8485269
28284265	159063100
	141421325
282842706	1764177500
	1697056236
	67121264

15. Represent $\sqrt{17}$ on number line. **[CBSE 2011]**

Sol. 17 can be written as $$17 = 16 + 1 = 4^2 + 1^2$$

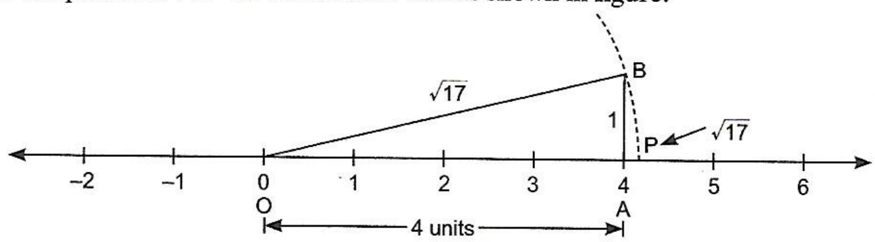
$\therefore$ $$\sqrt{17} = \sqrt{4^2 + 1^2}$$

$\Rightarrow$ $$OB = \sqrt{OA^2 + AB^2}$$

$\therefore$ On the number line, we mark OA = 4 units,

AB = 1 unit and AB $\perp$ OA at A.

Using a compass with centre O and radius OB, draw an arc intersecting the number line at the point P. Then point P corresponds to $\sqrt{17}$ on the number line as shown in figure.



16. Express 1.4191919... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **[CBSE 2010]**

Sol. Let $x = 1.4191919... = 1.4\overline{19}$

Multiplying both sides by 10, we get $10x = 14.\overline{19}$

Here, two digits are repeated continuously, therefore, again multiplying both sides by 100, we get

$$1000x = 1419.\overline{19} = 1405 + 14.\overline{19} = 1405 + 10x$$

$\Rightarrow$ $$1000x - 10x = 1405 \Rightarrow 990x = 1405$$

$\Rightarrow$ $$x = \frac{1405}{990} = \frac{281}{198}$$

17. In the following equations, examine whether x, y and z represents rational or irrational number.

(i) $x^3 = 27$ **(ii) $y^2 = 7$** **(iii) $z^2 = 0.16$**

Sol. (i) $$x^3 = 27$$

$\Rightarrow$ $$x = \sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3 = \frac{3}{1}$$

So, it is a rational number.

(ii) $$y^2 = 7$$

$\Rightarrow$ $$y = \sqrt{7} \neq \frac{p}{q}$$

So, it is an irrational number.

(iii) $$z^2 = 0.16 = \frac{16}{100}$$

$\therefore$ $$z = \sqrt{\frac{16}{100}} = \sqrt{\frac{4 \times 4}{10 \times 10}}$$

$$= \frac{4}{10} = \frac{2}{5} = \frac{p}{q}$$

Hence, it is a rational number.

18. State whether the following statements are true or false. Give reasons for your answers.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

Sol. (i) False, because whole numbers contains 0 but natural numbers does not, i.e. 0 is not a natural number.

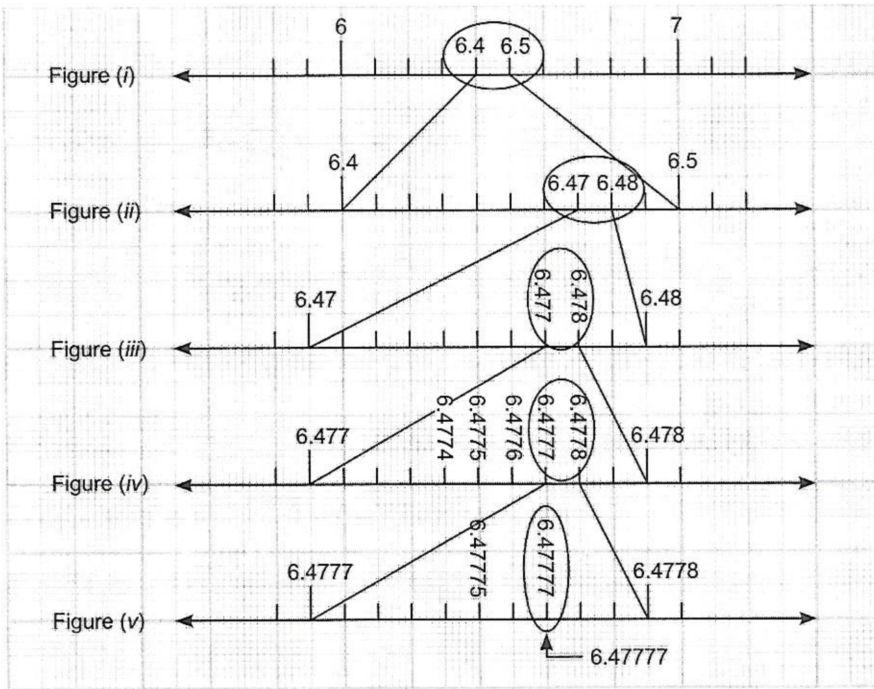
(ii) True, because every integer can be expressed in the form $\frac{p}{q}$, $q = 1$.

(iii) False, because $\frac{2}{5}$ is not an integer.

Long Answer Type Questions [4 Marks]

19. Visualise the representation of $6.4\overline{7}$ on the number line up to 5 decimal places, that is up to 6.47777. Draw figure only.

Sol.



20. Express $1.3\overline{2} + 0.\overline{35}$ as a fraction in simplest form. **[CBSE 2011]**

Sol. Let $x = 1.3\overline{2}$ and $y = 0.\overline{35}$

(*i*) Consider $x = 1.3\overline{2} = 1.32222...$

$$\Rightarrow \quad 10x = 13.222... = 13.\overline{2} \quad ...(i)$$

$$\Rightarrow \quad 100x = 132.\overline{2} \quad ...(ii)$$

Subtracting (*i*) from (*ii*), we get

$$100x - 10x = 132.\overline{2} - 13.\overline{2}$$

$$90x = 119$$

$$x = \frac{119}{90}$$

(*ii*) Consider $y = 0.\overline{35} = 0.353535...$ ...(*iii*)

$$\Rightarrow \quad 100y = 35.3535... = 35.\overline{35}$$

$$\therefore \quad 100y - y = 35.\overline{35} - 0.\overline{35} \quad ...(iv)$$

Subtracting (*iii*) from (*iv*), we get

$$99y = 35$$

$$y = \frac{35}{99}$$

Therefore,

$$1.3\overline{2} + 0.\overline{35} = x + y$$

$$= \frac{119}{90} + \frac{35}{99} = \frac{1309 + 350}{90 \times 11}$$

$$= \frac{1659}{90 \times 11} = \frac{553}{330}$$

PRACTICE QUESTIONS BASED ON EXERCISES 1.1, 1.2, 1.3 AND 1.4.

1. Check whether 3.142678 is a rational or an irrational number.
2. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$. **[CBSE 2011]**
3. Insert 3 rational numbers between $\frac{1}{5}$ and $\frac{3}{5}$.
4. Can we insert 100 rational numbers between 2 and 7? If yes, why?

5. Which whole number is not a natural number?
6. Justify 3.020020002 is a rational number.
7. Write two irrational numbers between $\frac{2}{5}$ and $\frac{3}{4}$.
8. Show that 5.672894 is a rational number.
9. Write 5 rational numbers and 5 irrational numbers in decimal form.
10. State four values of n for which $\sqrt{n}$ is a rational number.
11. Show that $0.4444... = 0.\bar{4}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
12. Express 1.2353535... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **[CBSE 2010]**
13. Show that $0.235555... = 0.23\bar{5}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
14. Find three different irrational numbers between the rational numbers 0.13 and 0.14.
15. Find a rational number and also an irrational number lying between the numbers 0.401001000100001... and 0.404004000400004... .
16. Find three rational numbers between 0.131331333133331... and 0.242442444244442... .
17. Identify the following as rational or irrational numbers:

    (i) $\sqrt{4}$ (ii) $\sqrt{\frac{9}{27}}$

    (iii) $\sqrt{1.44}$ (iv) $3\sqrt{18}$

    (v) 0.5918 (vi) $(7+\sqrt{2})-(4+\sqrt{2})$
18. Locate $\sqrt{13}$ on the number line.
19. Visualise $5.\overline{18}$ on the number line up to 4 decimal places.
20. Express $0.\overline{38} + 1.\overline{27}$ as a fraction in simplest form. **[CBSE 2015]**

Operations on Real Numbers

Properties of irrational numbers:

- Like rational numbers, irrational numbers also satisfies the commutative, associative and distributive laws for addition and multiplication.
- The sum, difference, quotients and products of two irrational numbers are not always irrational.

  (i) $\sqrt{13}+(-\sqrt{13}) = 0$ (Rational) (ii) $(3+\sqrt{2})+(3-\sqrt{2}) = 6$ (Rational)

  (iii) $\sqrt{5}-\sqrt{5} = 0$ (Rational) (iv) $\frac{\sqrt{12}}{\sqrt{12}} = \frac{2\sqrt{3}}{2\sqrt{3}} = 1$ (Rational)

  (v) $\sqrt{5}\times\sqrt{5} = 5$ (Rational)
- The sum or difference of a rational number and an irrational number is always an irrational number. For example:

  (i) $a+\sqrt{b}$ is an irrational number. (ii) $a-\sqrt{b}$ is an irrational number.
- The multiplication and division of a non-zero rational number with an irrational number is always irrational. For example:

  (i) $a\sqrt{b}$ is an irrational number. (ii) $a \div \sqrt{b}$ is an irrational number.

  (iii) $\sqrt{a} \div b$ is an irrational number.
- The multiplication and division of an irrational number by another irrational number results to a rational number. For example:

  (i) $5(\sqrt{3})^2 \div 4 = 5 \times 3 \div 4 = \frac{15}{4}$ (ii) $(\sqrt{3}-2)(\sqrt{3}+2) = (\sqrt{3})^2 - 2^2 = -1$

Operation of taking square roots of real numbers:

- **Surd:** Let $a > 0$ be a real number and n be a positive integer. Then $\sqrt[n]{a} = b$, if $b^n = a$ and $b > 0$. So, any number in the form $\sqrt[n]{a}$ and cannot be written as a rational number is called surd.
- The symbol $\sqrt{\ }$ is called radical sign.

- 'n' is known as order of surd and 'a' is known as radicand.
- Every surd is an irrational number, but every irrational number is not a surd.

Identities related to surds:

Let a and b be positive real numbers. Then the following identities holds:

(i) $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$ (iv) $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

(v) $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$ (vi) $(\sqrt{a} - \sqrt{b})^2 = a - 2\sqrt{ab} + b$

(vii) $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

Rationalisation: The process of reducing a given surd to a rational form after multiplying it by a suitable surd is known as rationalisation.

For example: To rationalise the denominator of $\frac{1}{\sqrt{a} + b}$, we multiply this by $\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} - \sqrt{b}}$, where a and b are integers.

To rationalise the denominator of $\frac{1}{\sqrt{a} - \sqrt{b}}$, we multiply this by $\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}}$, where a and b are integers.

The rationalising factor of $\frac{1}{a \pm \sqrt{b}}$ is $a \mp \sqrt{b}$.

SOLVED QUESTIONS BASED ON EXERCISE 1.5

Very Short Answer Type Questions [1 Mark]

1. Simplify $\sqrt{72} + \sqrt{800} - \sqrt{18}$. **[CBSE 2014]**

Sol. $\sqrt{72} + \sqrt{800} - \sqrt{18} = \sqrt{6 \times 6 \times 2} + \sqrt{2 \times 2 \times 2 \times 10 \times 10} - \sqrt{3 \times 3 \times 2}$

$$= 6\sqrt{2} + 20\sqrt{2} - 3\sqrt{2} = (6 + 20 - 3)\sqrt{2}$$

$$= 23\sqrt{2}$$

2. State with reasons whether $\sqrt{20} \times \sqrt{45}$ is a surd or not?

Sol. We have $\sqrt{20} \times \sqrt{45} = \sqrt{20 \times 45} = \sqrt{900}$

$$= \sqrt{30 \times 30} = 30,$$

which is a rational number and therefore $\sqrt{20} \times \sqrt{45}$ is not a surd.

3. Simplify $(\sqrt{13} + \sqrt{5})(\sqrt{13} - \sqrt{5})$.

Sol. $(\sqrt{13} + \sqrt{5})(\sqrt{13} - \sqrt{5}) = (\sqrt{13})^2 - (\sqrt{5})^2$ [$\because (a + b)(a - b) = a^2 - b^2$]

$$= 13 - 5 = 8$$

4. Simplify $\sqrt{125} \times \sqrt{5}$.

Sol. $\sqrt{125} \times \sqrt{5} = (5^3)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} = (5)^{\frac{3}{2}} \times (5)^{\frac{1}{2}}$

$$= (5)^{\frac{3}{2} + \frac{1}{2}}$$ [$a^m . a^n = a^{m+n}$]

$$= 5^{\frac{4}{2}} = 5^2 = 25$$

Short Answer Type Questions I [2 Marks]

5. Find the value of $\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}}$, if $\sqrt{3} = 1.73$. **[CBSE 2015]**

Sol. Consider $\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}}$, on rationalising the denominator by multiplying and dividing it by $\sqrt{2+\sqrt{3}}$, we get

$$\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} \times \sqrt{\frac{2+\sqrt{3}}{2+\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}}$$

$$= \frac{2+\sqrt{3}}{1} = 2 + \sqrt{3} = 2 + 1.73 = 3.73$$

6. Simplify $\dfrac{6-4\sqrt{3}}{6+4\sqrt{3}}$ by rationalising the denominator. **[CBSE 2014]**

Sol. Here, the denominator is $6 + 4\sqrt{3}$.

Multiplying the numerator and denominator by its conjugate $(6 - 4\sqrt{3})$, we get

$$\frac{6-4\sqrt{3}}{6+4\sqrt{3}} = \left(\frac{6-4\sqrt{3}}{6+4\sqrt{3}}\right) \times \left(\frac{6-4\sqrt{3}}{6-4\sqrt{3}}\right) = \frac{(6-4\sqrt{3})^2}{(6)^2-(4\sqrt{3})^2}$$

$$= \frac{36-48\sqrt{3}+48}{36-48} \qquad [(a-b)^2 = a^2 - 2ab + b^2]$$

$$= \frac{84-48\sqrt{3}}{-12} = \frac{12(7-4\sqrt{3})}{-12} = 4\sqrt{3} - 7$$

7. If $x = 3 + 2\sqrt{2}$, then find whether $x + \dfrac{1}{x}$ is rational or irrational. **[CBSE 2011]**

Sol. Given $x = 3+2\sqrt{2}$

$$\therefore \quad \frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$\therefore \quad x+\frac{1}{x} = 3+2\sqrt{2}+3-2\sqrt{2} = 6$$

which is a rational number. Hence, $x + \dfrac{1}{x}$ for $x = 3+2\sqrt{2}$ is a rational number.

8. Simplify $\sqrt[4]{81} - 8(\sqrt[3]{216}) + 15(\sqrt[5]{32}) + \sqrt{225}$. **[CBSE 2010]**

Sol. Here,

$$\sqrt[4]{81} = (81)^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 3^{4\times\frac{1}{4}} = 3$$

$$\sqrt[3]{216} = (216)^{\frac{1}{3}} = (6^3)^{\frac{1}{3}} = 6^{3\times\frac{1}{3}} = 6$$

$$\sqrt[5]{32} = (32)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2$$

$$\sqrt{225} = (225)^{\frac{1}{2}} = (15^2)^{\frac{1}{2}} = 15^{2\times\frac{1}{2}} = 15$$

Hence, $\sqrt[4]{81} - 8(\sqrt[3]{216}) + 15(\sqrt[5]{32}) + \sqrt{225} = 3 - 8 \times 6 + 15 \times 2 + 15 = 3 - 48 + 30 + 15 = 48 - 48 = 0$

9. Find the value of a and b, if $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$.

Sol. Here, the denominator is $\sqrt{3} + 1$.

Multiplying the numerator and denominator by its conjugate $\sqrt{3} - 1$, we get

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) \times \left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - 1^2} = \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = \frac{2(2-\sqrt{3})}{2} = 2-\sqrt{3}$$

$\therefore \quad 2-\sqrt{3} = a + b\sqrt{3}$

Hence, on equating rational and irrational part both sides, we get $a = 2, b = -1$.

10. If $a = \sqrt{2}+1$, find the value of $\left(a - \frac{1}{a}\right)^2$.

Sol. Given $\quad a = \sqrt{2}+1$

$$\therefore \quad \frac{1}{a} = \frac{1}{\sqrt{2}+1}$$

$$\Rightarrow \quad \frac{1}{a} = \left(\frac{1}{\sqrt{2}+1}\right)\times\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$$ (Rationalising the denominator)

$$= \frac{\sqrt{2}-1}{(\sqrt{2})^2-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

$$\therefore \quad a - \frac{1}{a} = (\sqrt{2}+1)-(\sqrt{2}-1)$$

$$= \sqrt{2}+1-\sqrt{2}+1 = 2$$

$$\therefore \quad \left(a-\frac{1}{a}\right)^2 = 2^2 = 4$$

Short Answer Type Questions II [3 Marks]

11. Simplify $\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$ **[CBSE 2010]**

Sol.
$$\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}} = \left(\frac{4+\sqrt{5}}{4-\sqrt{5}}\right)\times\left(\frac{4+\sqrt{5}}{4+\sqrt{5}}\right) + \left(\frac{4-\sqrt{5}}{4+\sqrt{5}}\right)\times\left(\frac{4-\sqrt{5}}{4-\sqrt{5}}\right)$$

(Rationalising both denominators)

$$= \frac{(4+\sqrt{5})^2}{(4)^2-(\sqrt{5})^2} + \frac{(4-\sqrt{5})^2}{(4)^2-(\sqrt{5})^2} = \frac{16+5+8\sqrt{5}}{16-5} + \frac{16+5-8\sqrt{5}}{16-5}$$

$$= \frac{1}{11}[21+8\sqrt{5}+21-8\sqrt{5}] = \frac{42}{11}$$

12. Simplify $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$.

Sol. Given $3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50}$

Now,
$$\sqrt{45} = \sqrt{5\times\underline{3\times 3}} = 3\sqrt{5}$$
$$\sqrt{125} = \sqrt{\underline{5\times 5}\times 5} = 5\sqrt{5}$$
$$\sqrt{200} = \sqrt{2\times\underline{10\times 10}} = 10\sqrt{2}$$
$$\sqrt{50} = \sqrt{\underline{5\times 5}\times 2} = 5\sqrt{2}$$

$$\therefore \quad 3\sqrt{45} - \sqrt{125} + \sqrt{200} - \sqrt{50} = 3\times 3\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2} = 9\sqrt{5} - 5\sqrt{5} + 5\sqrt{2} = 4\sqrt{5} + 5\sqrt{2}$$

13. **If $p = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $q = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then find $p^2 + q^2$.** **[CBSE 2011]**

Sol. Given $p = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Multiplying and dividing R.H.S. by $\sqrt{3}-\sqrt{2}$, we get

$$p = \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)\times\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) = \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2-2\sqrt{6}}{3-2}$$

$$= 5-2\sqrt{6}$$

and $$q = \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)\times\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$$ (Rationalising the denominator)

On solving, we get $q = 5+2\sqrt{6}$

Now, $pq = (5-2\sqrt{6})(5+2\sqrt{6}) = (5)^2-(2\sqrt{6})^2 = 25-24 = 1$

and $p+q = 5-2\sqrt{6}+5+2\sqrt{6} = 10$

Now, $(p+q)^2 = 10^2$

$\Rightarrow$ $p^2+q^2+2pq = 100 \Rightarrow p^2+q^2+2\times 1 = 100$

$\Rightarrow$ $p^2+q^2 = 100-2 = 98$

14. **If $x = 2+\sqrt{3}$, find the value of $x^3 + \dfrac{1}{x^3}$.** **[CBSE 2015]**

Sol. Given $x = 2+\sqrt{3}$

$\therefore$ $$\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \left(\frac{1}{2+\sqrt{3}}\right)\times\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$$ (Rationalising the denominator)

$\Rightarrow$ $$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$\therefore$ $$x+\frac{1}{x} = 2+\sqrt{3}+2-\sqrt{3} = 4$$

$\Rightarrow$ $$\left(x+\frac{1}{x}\right)^3 = 4^3$$ & cubing on both sides

$\Rightarrow$ $$x^3+\frac{1}{x^3}+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right) = 64 \Rightarrow x^3+\frac{1}{x^3}+3\times 4 = 64$$

$\Rightarrow$ $$x^3+\frac{1}{x^3} = 64-12 \Rightarrow x^3+\frac{1}{x^3} = 52$$

15. **If $\sqrt{5} = 2.236$ and $\sqrt{6} = 2.449$, find the value of $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$.** **[CBSE 2016]**

Sol. Let $$x = \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\right)\times\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)$$ (Rationalising the denominator)

$\Rightarrow$ $$x = \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{5-3}$$

$$= \frac{1}{2}(\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6})$$

Again, let $$y = \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

Similarly, on rationalising the denominator and solving, we get

$$y = \frac{1}{2}(\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6})$$

$$\therefore \quad x+y = \frac{1}{2}[\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}]$$

$$= \frac{1}{2}[2\sqrt{5}-2\sqrt{6}] = \sqrt{5}-\sqrt{6}$$

$$= 2.236 - 2.449 = -0.213$$

16. Represent $\sqrt{10.5}$ on the number line. **[CBSE 201**

Sol. Steps of Construction:

(*i*) Draw a line AB such that AB = 10.5 units on the number line.

(*ii*) Extend the line *l* further from B up to C such that BC = 1 unit.

(*iii*) Find the mid-point of AC and mark it as O.

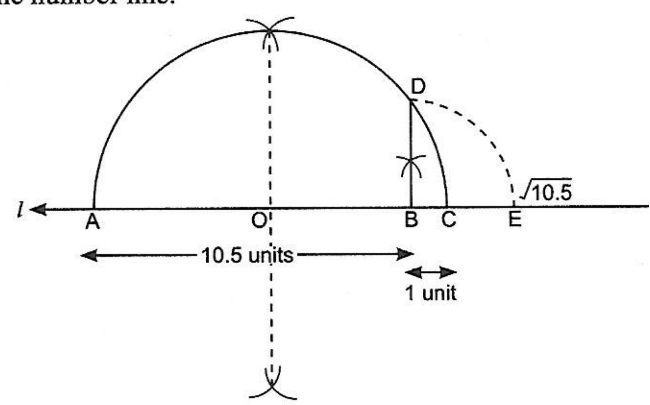
(*iv*) Draw a semicircle with centre O and radius OC.

(*v*) Draw a line perpendicular to AC passing through point B and cut the semicircle at D.

(*vi*) Taking B as centre, draw an arc of radius BD which intersects the number line at E.

(*vii*) Point E represents $\sqrt{10.5}$ on the number line.

$\therefore$ BD = BE = $\sqrt{10.5}$ units, with B as zero.



17. Simplify $3\sqrt{45}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}$.

Sol.

$$3\sqrt{45}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3} = 3\sqrt{\underline{3\times3}\times5}-\frac{5}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}+4\sqrt{3}$$

$$= 9\sqrt{5}-\frac{5}{6}\sqrt{3}+4\sqrt{3}$$

$$= 9\sqrt{5}+\left(4-\frac{5}{6}\right)\sqrt{3} = 9\sqrt{5}+\frac{19}{6}\sqrt{3}$$

Long Answer Type Questions [4 Marks]

18. Evaluate $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ when it is given that $\sqrt{5}$ = 2.2 and $\sqrt{10}$ = 3.2. **[CBSE 20**

Sol. Consider the denominator $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$$= \sqrt{10}+\sqrt{5\times\underline{2\times2}}+\sqrt{\underline{2\times2}\times2\times5}-\sqrt{5}-\sqrt{\underline{4\times4}\times5}$$

$$= \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$$

$$= 3\sqrt{10}+2\sqrt{5}-5\sqrt{5} = 3\sqrt{10}-3\sqrt{5} = 3(\sqrt{10}-\sqrt{5})$$

$$\therefore \quad \frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}} = \frac{15}{3(\sqrt{10}-\sqrt{5})} = \frac{5}{\sqrt{10}-\sqrt{5}}$$

Multiplying and dividing by the conjugate of $\sqrt{10}-\sqrt{5}$, i.e. $\sqrt{10}+\sqrt{5}$, we get

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}} = \left(\frac{5}{\sqrt{10}-\sqrt{5}}\right)\times\left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}\right)$$

$$= \frac{5(\sqrt{10}+\sqrt{5})}{(\sqrt{10})^2-(\sqrt{5})^2} \qquad [(a+b)(a-b)=a^2-b^2]$$

$$= \frac{5(\sqrt{10}+\sqrt{5})}{10-5} = \sqrt{10}+\sqrt{5} = 3.2+2.2 = 5.4$$

19. If $a = \dfrac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}}$ and $b = \dfrac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}$, then show that $\sqrt{a}-\sqrt{b}-2\sqrt{ab} = 0$. **[CBSE 2014]**

Sol.

$$\sqrt{a} = \sqrt{\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}}}$$

$$= \sqrt{\left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}}\right)}\times\sqrt{\left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}\right)} \qquad \text{(Rationalising the denominator)}$$

$$= \sqrt{\frac{(\sqrt{10}+\sqrt{5})^2}{(\sqrt{10})^2-(\sqrt{5})^2}} \qquad [\because (a-b)(a+b)=a^2-b^2]$$

$$= \frac{\sqrt{10}+\sqrt{5}}{\sqrt{10-5}}$$

$$\therefore \quad \sqrt{a} = \frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}}$$

Similarly,

$$\sqrt{b} = \sqrt{\frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}}$$

After rationalising the denominator, we get

$$\sqrt{b} = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}$$

and

$$\sqrt{a.b} = \sqrt{a}\times\sqrt{b} = \left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}}\right)\times\left(\frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}\right) = \frac{(\sqrt{10})^2-(\sqrt{5})^2}{(\sqrt{5})^2} = \frac{10-5}{5} = \frac{5}{5} = 1$$

$$\therefore \quad \text{L.H.S.} = \sqrt{a}-\sqrt{b}-2\sqrt{ab} = \left(\frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}}\right)-\left(\frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}\right)-2\times 1$$

$$= \frac{1}{\sqrt{5}}(\sqrt{10}+\sqrt{5}-\sqrt{10}+\sqrt{5})-2 = \frac{2\sqrt{5}}{\sqrt{5}}-2 = 2-2$$

$$= 0 = \text{R.H.S.}$$

Hence proved.

20. If $x = \dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ and $y = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$, find the value of x^2+y^2+xy. **[CBSE 2014]**

Sol. Consider

$$x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$= \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\times\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right) \qquad \text{(Rationalising the denominator)}$$

$$= \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2-1^2} = \frac{2+1+2\sqrt{2}}{2-1} = 3+2\sqrt{2}$$

Similarly, $y = 3-2\sqrt{2}$

Now, $xy = (3+2\sqrt{2})(3-2\sqrt{2}) = (3)^2-(2\sqrt{2})^2 = 9-8 = 1$

and $x+y = (3+2\sqrt{2})+(3-2\sqrt{2}) = 6$

Squaring both sides, we get

$$(x+y)^2 = 36 \Rightarrow x^2+y^2+2xy = 36$$

$$\Rightarrow \quad x^2+y^2+2\times 1 = 36 \Rightarrow x^2+y^2 = 34$$

Hence, $x^2+y^2+xy = 34+1 = 35$

21. Simplify $\sqrt{\dfrac{\sqrt{20}+\sqrt{11}}{\sqrt{20}-\sqrt{11}}}$. **[CBSE 2014]**

Sol. Rationalising the denominator, we get

$$\sqrt{\frac{\sqrt{20}+\sqrt{11}}{\sqrt{20}-\sqrt{11}}} = \sqrt{\frac{\sqrt{20}+\sqrt{11}}{\sqrt{20}-\sqrt{11}}}\times\sqrt{\frac{\sqrt{20}+\sqrt{11}}{\sqrt{20}+\sqrt{11}}}$$

$$= \sqrt{\frac{(\sqrt{20}+\sqrt{11})^2}{(\sqrt{20})^2-(\sqrt{11})^2}} = \frac{\sqrt{20}+\sqrt{11}}{\sqrt{20-11}} = \frac{\sqrt{20}+\sqrt{11}}{\sqrt{9}} = \frac{1}{3}(\sqrt{20}+\sqrt{11})$$

22. If $x+\dfrac{1}{x} = \sqrt{3}$, find the value of $x^3+\dfrac{1}{x^3}$. **[CBSE 2016]**

Sol. Given $x+\dfrac{1}{x} = \sqrt{3}$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^3 = (\sqrt{3})^3 \qquad \text{(Cubing both sides)}$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3x.\frac{1}{x}\left(x+\frac{1}{x}\right) = 3^{\frac{3}{2}} \Rightarrow x^3+\frac{1}{x^3}+3\sqrt{3} = \sqrt{3^3} = \sqrt{3\times3\times3}$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3\sqrt{3} = 3\sqrt{3} \Rightarrow x^3+\frac{1}{x^3} = 3\sqrt{3}-3\sqrt{3} \Rightarrow x^3+\frac{1}{x^3} = 0$$

23. If $a = 7-4\sqrt{3}$, find the value of $\sqrt{a}+\dfrac{1}{\sqrt{a}}$. **[CBSE 2011, 2014, 2016; HOTS]**

Sol. Given $a = 7-4\sqrt{3}$

$$\therefore \quad \frac{1}{a} = \frac{1}{7-4\sqrt{3}}$$

$$= \left(\frac{1}{7-4\sqrt{3}}\right)\times\left(\frac{7+4\sqrt{3}}{7+4\sqrt{3}}\right) \qquad \text{(Rationalising the denominator)}$$

$$\Rightarrow \quad \frac{1}{a} = \frac{7+4\sqrt{3}}{(7)^2-(4\sqrt{3})^2} \qquad [\because (a-b)(a+b) = a^2-b^2]$$

$$= \frac{7+4\sqrt{3}}{49-48} = 7+4\sqrt{3}$$

$$\therefore \quad a+\frac{1}{a} = 7-4\sqrt{3}+7+4\sqrt{3} = 14$$

Now, $$\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)^2 = a+\frac{1}{a}+2.a.\frac{1}{a} = 14+2 = 16$$

$$\therefore \quad \sqrt{a}+\frac{1}{\sqrt{a}} = \sqrt{16} = 4$$

24. Prove that $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5$. **[CBSE 2011, 2015]**

Sol. $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$

$$=\left[\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}\right]-\left[\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}\right]+\left[\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}\right]$$
$$-\left[\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right]+\left[\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}\right]$$

$$=\left[\frac{3+\sqrt{8}}{9-8}\right]-\left[\frac{\sqrt{8}+\sqrt{7}}{8-7}\right]+\left[\frac{\sqrt{7}+\sqrt{6}}{7-6}\right]-\left[\frac{\sqrt{6}+\sqrt{5}}{6-5}\right]+\left[\frac{\sqrt{5}+2}{5-4}\right]$$

$$=[3+\sqrt{8}]-[\sqrt{8}+\sqrt{7}]+[\sqrt{7}+\sqrt{6}]-[\sqrt{6}+\sqrt{5}]+[\sqrt{5}+2]$$

$$=3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2=5$$

25. Rationalise the denominator of $\frac{4}{2+\sqrt{3}+\sqrt{7}}$. **[CBSE 2011]**

Sol. $\frac{4}{2+\sqrt{3}+\sqrt{7}}=\frac{4}{(2+\sqrt{3})+\sqrt{7}}\times\frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}}=\frac{4(2+\sqrt{3}-\sqrt{7})}{(2+\sqrt{3})^2-(\sqrt{7})^2}$ [Using $(a+b)(a-b)=a^2-b^2$]

$=\frac{4(2+\sqrt{3}-\sqrt{7})}{4+3+4\sqrt{3}-7}$ [Using $(a+b)^2=a^2+b^2+2ab$]

$$=\frac{4(2+\sqrt{3}-\sqrt{7})}{7+4\sqrt{3}-7}=\frac{4(2+\sqrt{3}-\sqrt{7})}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{\sqrt{3}(2+\sqrt{3}-\sqrt{7})}{3}=\frac{2\sqrt{3}+3-\sqrt{21}}{3}=\frac{1}{3}[3+2\sqrt{3}-\sqrt{21}]$$

PRACTICE QUESTIONS BASED ON EXERCISE 1.5

1. Rationalise the denominator of $\frac{1}{\sqrt{5}}$.
2. Simplify $(\sqrt{3}-\sqrt{7})^2$.
3. Simplify $(\sqrt{3}+\sqrt{5})(3-\sqrt{5})$.
4. Rationalise the denominator of $\frac{5}{\sqrt{7}-\sqrt{5}}$.
5. Check whether $-5+2\sqrt{5}-\sqrt{5}$ is an irrational or a rational number. **[CBSE 2010]**
6. Find the value of $\frac{1}{\sqrt{10}}$, where $\sqrt{10}=3.162$. **[CBSE 2010]**
7. Simplify $8\sqrt{242}-5\sqrt{50}+3\sqrt{98}$.
8. Simplify $3\sqrt[3]{40}-4\sqrt[3]{320}$.
9. If $x=\sqrt{2}+1$, find the value of $x+\frac{1}{x}$. **[CBSE 2014]**
10. If $x=5\sqrt{2}-7$, find the value of $x^3-\frac{1}{x^3}$.
11. Simplify $\frac{(\sqrt{3}-\sqrt{5})(\sqrt{5}+\sqrt{3})}{7-2\sqrt{5}}$. **[CBSE 2016]**
12. Represent $\sqrt{6.5}$ on the number line.
13. Find the value of x and y, if $\frac{\sqrt{2}}{3\sqrt{6}-\sqrt{5}}=x\sqrt{3}-y\sqrt{10}$.
14. Find the value of a and b, if $\frac{2-\sqrt{5}}{2+3\sqrt{5}}=\sqrt{5}\,a+b$.
15. If $\sqrt{2}=1.4142$, then simplify $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$.
16. Find x^2, if $x=\frac{\sqrt{\sqrt{6}+2}+\sqrt{\sqrt{6}-2}}{\sqrt{\sqrt{6}+\sqrt{2}}}$. **[HOTS]**
17. If $a=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find the value of a^2+b^2-5ab. **[CBSE 2011]**

18. Rationalise the denominator of $\frac{3}{\sqrt{3}+\sqrt{5}-\sqrt{2}}$.

[CBSE 2015]

19. Prove that

$$\frac{1}{3+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+1}=1.$$

[CBSE 2011]

20. If $x=(2+\sqrt{5})^{\frac{1}{2}}+(2-\sqrt{5})^{\frac{1}{2}}$ and $y=(2+\sqrt{5})^{\frac{1}{2}}-(2-\sqrt{5})^{\frac{1}{2}}$, then evaluate x^2+y^2.

[CBSE 2011; HOTS]

Laws of Exponents for Real Numbers

- **Laws of Exponents:** Let m and n be exponents (powers) of base 'a' and $a>0$. Then

(i) $a^m . a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $\frac{a^m}{a^n}=a^{m-n}, m>n \left[\because \frac{1}{a^n}=a^{-n}\right]$

(iv) $a^{-m}=\frac{1}{a^m}$ (v) $a^0=1$ (vi) $a^m b^m=(ab)^m$

(vii) $(a^m)^{-n}=a^{-mn}$ (viii) $\frac{a^{-m}}{a^n}=a^{-m-n}=a^{-(m+n)}$ (ix) $a^{-m}\times a^{-n}=a^{m+n}$

Here, the base is a positive real number and the exponents are rational numbers.

- Let $a>0$ be a real number. Let p and q be integers such that p and q have no common factor other than 1 and $q>0$.

Then, $(a)^{\frac{p}{q}}=(a^p)^{\frac{1}{q}}=\sqrt[q]{a^p}$ or $a^{\frac{p}{q}}=\left(a^{\frac{1}{q}}\right)^p=(\sqrt[q]{a})^p$

So, both operations are possible.

- **Extended laws of exponents:** Let $a>0$ be a real number and m and n be rational numbers. Then, we have

(i) $(\sqrt[n]{a})^n=a$ (ii) $\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}$ [both a and b should be non-negative integer]

(iii) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$ (iv) $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=\sqrt[n]{\sqrt[m]{a}}$

(v) $\frac{\sqrt[p]{a^n}}{\sqrt[p]{a^m}}=\sqrt[p]{a^{n-m}}$ (vi) $\sqrt[p]{a^n\times a^m}=\sqrt[p]{a^{n+m}}$ (vii) $\sqrt[p]{(a^n)^m}=\sqrt[p]{a^{n.m}}$

SOLVED QUESTIONS BASED ON EXERCISE 1.6

Very Short Answer Type Questions [1 Mark]

1. Simplify $\left(\sqrt{x^3}\right)^{\frac{2}{3}}$ **[CBSE 2016]**

Sol.

$$\left(\sqrt{x^3}\right)^{\frac{2}{3}}=\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} \qquad \left[\sqrt{x^3}=(x^3)^{\frac{1}{2}}\right]$$

$$=x^{\frac{3}{2}\times\frac{2}{3}} \qquad \left[(x^m)^n=x^{mn}\right]$$

$$=x$$

$$\therefore \quad \left(\sqrt{x^3}\right)^{\frac{2}{3}}=x$$

2. Find the value of $\frac{2^\circ+7^\circ}{5^\circ}$. **[CBSE 2011]**

Sol. We know that

$$a^\circ=1$$

$$\therefore \quad \frac{2^\circ+7^\circ}{5^\circ}=\frac{1+1}{1}=\frac{2}{1}=2$$

3. Find the value of $\sqrt{(3^{-2})}$. **[CBSE 2011]**

Sol.

$$\sqrt{(3^{-2})} = (3^{-2})^{\frac{1}{2}}$$
$$= 3^{-2\times\frac{1}{2}} \qquad [(a^m)^n = a^{mn}]$$
$$= 3^{-1}$$
$$= \frac{1}{3} \qquad \left[a^{-m} = \frac{1}{a^m}\right]$$

4. Simplify $16^{-\frac{1}{4}} \times \sqrt[4]{16}$. **[CBSE 2010]**

Sol.

$$16^{-\frac{1}{4}} \times \sqrt[4]{16} = 16^{-\frac{1}{4}} \times 16^{\frac{1}{4}}$$
$$= 16^{-\frac{1}{4}+\frac{1}{4}} \qquad [a^m.a^n = a^{m+n}]$$
$$= 16^0 \qquad [a^0 = 1]$$
$$= 1$$

5. Which is the greatest among $\sqrt{2}, \sqrt[3]{4}$ and $\sqrt[4]{3}$? **[CBSE 2015]**

Sol. The order of the given surds are 2, 3 and 4 respectively.

$\therefore$ L.C.M. of 2, 3 and 4 = 12

Now,

$$\sqrt{2} = \sqrt[12]{2^6} = \sqrt[12]{64}$$
$$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$$
$$\sqrt[4]{3} = \sqrt[12]{3^3} = \sqrt[12]{27}$$

Clearly, $256 > 64 > 27$

$\Rightarrow$ $\sqrt[12]{256} > \sqrt[12]{64} > \sqrt[12]{27}$

$\Rightarrow$ $\sqrt[3]{4} > \sqrt{2} > \sqrt[4]{3}$

6. Find the value of $(81)^{0.16} \times (81)^{0.09}$. **[CBSE 2015]**

Sol.

$$(81)^{0.16} \times (81)^{0.09} = (81)^{0.16+0.09} \qquad [a^m.a^n = a^{m+n}]$$
$$= (81)^{0.25} = (81)^{\frac{25}{100}} = (3^4)^{\frac{1}{4}} = 3^{4\times\frac{1}{4}} = 3$$

7. Find the value of $x^{a-b} \times x^{b-c} \times x^{c-a}$. **[CBSE 2016]**

Sol.

$$x^{a-b} \times x^{b-c} \times x^{c-a} = x^{a-b+b-c+c-a} \qquad [a^m.a^n.a^p = a^{m+n+p}]$$
$$= x^0 = 1 \qquad [a^0 = 1]$$

8. Find the value of $\left[(16)^{\frac{1}{2}}\right]^{\frac{1}{2}}$. **[CBSE 2016]**

Sol.

$$\left[(16)^{\frac{1}{2}}\right]^{\frac{1}{2}} = \left[(4^2)^{\frac{1}{2}}\right]^{\frac{1}{2}} = \left(4^{2\times\frac{1}{2}}\right)^{\frac{1}{2}} \qquad [(a^m)^n = a^{mn}]$$
$$= (4)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^{2\times\frac{1}{2}} = 2$$

9. Find the value of $\left(\frac{64}{25}\right)^{-\frac{3}{2}}$. **[CBSE 2016]**

Sol. $$\left(\frac{64}{25}\right)^{-\frac{3}{2}} = \left[\left(\frac{8^2}{5^2}\right)\right]^{-\frac{3}{2}} = \left[\left(\frac{8}{5}\right)^2\right]^{-\frac{3}{2}} = \left(\frac{8}{5}\right)^{-2\times\frac{3}{2}} = \left(\frac{8}{5}\right)^{-3} = \left(\frac{5}{8}\right)^3 = \frac{125}{512}$$

10. Simplify $3^{\frac{2}{3}} \cdot 3^{\frac{1}{5}}$. **[CBSE 2016]**

Sol.

$$3^{\frac{2}{3}} \cdot 3^{\frac{1}{5}} = 3^{\frac{2}{3}+\frac{1}{5}} \qquad [a^m.a^n = a^{m+n}]$$
$$= 3^{\frac{10+3}{15}} = 3^{\frac{13}{15}}$$

11. Simplify $\dfrac{7^{\frac{1}{3}}}{7^{\frac{1}{5}}}$.

Sol. $\dfrac{7^{\frac{1}{3}}}{7^{\frac{1}{5}}} = 7^{\left(\frac{1}{3}-\frac{1}{5}\right)} = 7^{\frac{5-3}{15}} = 7^{\frac{2}{15}}$

Short Answer Type Questions I [2 Marks]

12. If $z = 0.064$, then find the value of $\left(\dfrac{1}{z}\right)^{\frac{1}{3}}$. [CBSE 2013]

Sol. Given $z = 0.064$

$\therefore$ $$\frac{1}{z} = \frac{1}{0.064} = \frac{1000}{64} = \left(\frac{10}{4}\right)^3$$

So, $$\left(\frac{1}{z}\right)^{\frac{1}{3}} = \left[\left(\frac{10}{4}\right)^3\right]^{\frac{1}{3}} = \left(\frac{10}{4}\right)^{3\times\frac{1}{3}}$$ $[(a^m)^n = a^{mn}]$

$$= \frac{10}{4} = \frac{5}{2} = 2.5$$

13. Simplify $\left[\dfrac{15^{\frac{1}{4}}}{9^{\frac{1}{4}}}\right]$. [CBSE 2011]

Sol. $$\left[\frac{15^{\frac{1}{4}}}{9^{\frac{1}{4}}}\right] = \frac{(5\times 3)^{\frac{1}{4}}}{(3^2)^{\frac{1}{4}}} = \frac{5^{\frac{1}{4}}\times 3^{\frac{1}{4}}}{3^{2\times\frac{1}{4}}}$$

$$= 5^{\frac{1}{4}}\times 3^{\frac{1}{4}-\frac{1}{2}}$$ $\left[\dfrac{a^m}{a^n} = a^{m-n}\right]$

$$= 5^{\frac{1}{4}}\times 3^{\frac{1-2}{4}} = 5^{\frac{1}{4}}\times 3^{-\frac{1}{4}}$$

$$= \left(\frac{5}{3}\right)^{\frac{1}{4}}$$ $\left[a^{-m} = \dfrac{1}{a^m}\right]$

14. Evaluate $\left(\dfrac{32}{243}\right)^{-\frac{4}{5}}$. [CBSE 2011]

Sol. $$\left(\frac{32}{243}\right)^{-\frac{4}{5}} = \left[\frac{2^5}{3^5}\right]^{-\frac{4}{5}} = \left[\left(\frac{2}{3}\right)^5\right]^{-\frac{4}{5}}$$

$$= \left(\frac{2}{3}\right)^{-5\times\frac{4}{5}}$$ $[(a^m)^n = a^{mn}]$

$$= \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$$ $\left[a^{-m} = \dfrac{1}{a^m}\right]$

$$= \frac{81}{16}$$

15. Simplify $\sqrt[4]{\sqrt[3]{x^2}}$ and express the result in the exponent form of x. [CBSE 2011]

Sol. Given $\sqrt[4]{\sqrt[3]{x^2}}$

The given expression can be written as

$$\sqrt[4]{\sqrt[3]{x^2}} = \left[(x^2)^{\frac{1}{3}}\right]^{\frac{1}{4}} = \left(x^{2\times\frac{1}{3}}\right)^{\frac{1}{4}}$$ $[(a^m)^n = a^{mn}]$

$$= x^{2\times\frac{1}{3}\times\frac{1}{4}} = x^{\frac{1}{6}}$$

16. Find the value of $\dfrac{4}{(216)^{-\frac{2}{3}}} - \dfrac{1}{(256)^{-\frac{3}{4}}}$. **[CBSE 2011]**

Sol. Here, $(216)^{-\frac{2}{3}} = (6^3)^{-\frac{2}{3}} = 6^{-3\times\frac{2}{3}} = 6^{-2} = \dfrac{1}{6^2} = \dfrac{1}{36}$

and $\dfrac{1}{(256)^{-\frac{3}{4}}} = (256)^{\frac{3}{4}} = (4^4)^{\frac{3}{4}} = 4^{4\times\frac{3}{4}} = 4^3 = 64$

$\therefore \quad \dfrac{4}{(216)^{-\frac{2}{3}}} - \dfrac{1}{(256)^{-\frac{3}{4}}} = \dfrac{4}{\frac{1}{36}} - 64 = 4 \times 36 - 64 = 144 - 64 = 80$

17. Simplify: (*i*) $\left\{\left[(625)^{-\frac{1}{2}}\right]^{-\frac{1}{4}}\right\}^2$ (*ii*) $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right]$

Sol. (*i*) $\left\{\left[(625)^{-\frac{1}{2}}\right]^{-\frac{1}{4}}\right\}^2 = (625)^{\left(-\frac{1}{2}\right)\times\left(-\frac{1}{4}\right)\times 2} = (5^4)^{\frac{1}{4}} = 5^{4\times\frac{1}{4}} = 5$

(*ii*) $64^{-\frac{1}{3}} \times \left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right] = \left(4^3\right)^{-\frac{1}{3}} \times \left[\left(4^3\right)^{\frac{1}{3}} - \left(4^3\right)^{\frac{2}{3}}\right] = 4^{-3\times\frac{1}{3}} \times \left[4^{3\times\frac{1}{3}} - 4^{3\times\frac{2}{3}}\right]$

$= 4^{-1}[4 - 4^2] = \dfrac{1}{4}[4 - 16] = -\dfrac{12}{4} = -3$

Short Answer Type Questions II [3 Marks]

18. Find the value of $\dfrac{4}{(216)^{-\frac{2}{3}}} + \dfrac{1}{(256)^{-\frac{3}{4}}} + \dfrac{2}{(243)^{-\frac{1}{5}}}$.

Sol. $\dfrac{4}{(216)^{-\frac{2}{3}}} + \dfrac{1}{(256)^{-\frac{3}{4}}} + \dfrac{2}{(243)^{-\frac{1}{5}}} = \dfrac{4}{(6^3)^{\frac{-2}{3}}} + \dfrac{1}{(2^8)^{-\frac{3}{4}}} + \dfrac{2}{(3^5)^{-\frac{1}{5}}}$

$= \dfrac{4}{6^{-3\times\frac{2}{3}}} + \dfrac{1}{2^{-8\times\frac{3}{4}}} + \dfrac{2}{3^{-5\times\frac{1}{5}}} = \dfrac{4}{6^{-2}} + \dfrac{1}{2^{-6}} + \dfrac{2}{3^{-1}}$

$= 4 \times 6^2 + 2^6 + 2 \times 3 = 4 \times 36 + 64 + 6$

$= 144 + 70 = 214$

19. Prove that $\dfrac{2^{30} + 2^{29} + 2^{28}}{2^{31} + 2^{30} - 2^{29}} = \dfrac{7}{10}$. **[CBSE 2011]**

Sol. L.H.S. $= \dfrac{2^{30} + 2^{29} + 2^{28}}{2^{31} + 2^{30} - 2^{29}} = \dfrac{2^{28+2} + 2^{28+1} + 2^{28}}{2^{29+2} + 2^{29+1} - 2^{29}}$

$= \dfrac{2^{28}.2^2 + 2^{28}.2 + 2^{28}.1}{2^{29}.2^2 + 2^{29}.2 - 2^{29}.1} = \dfrac{2^{28}(2^2 + 2 + 1)}{2^{29}(2^2 + 2 - 1)}$

$= \dfrac{4 + 2 + 1}{2^{29-28}(4 + 2 - 1)}$ $\left[a^m = \dfrac{1}{a^{-m}} \text{ and } a^m . a^{-n} = a^{m-n}\right]$

$= \dfrac{7}{2(5)} = \dfrac{7}{10}$ = R.H.S. Hence proved.

20. Simplify $\left[5\,[8^{\frac{1}{3}}+27^{\frac{1}{3}}]\right]^{\frac{1}{4}}$. **[CBSE 2011]**

Sol.
$$\left[5\,[8^{\frac{1}{3}}+27^{\frac{1}{3}}]\right]^{\frac{1}{4}} = \left[5\left[(2^3)^{\frac{1}{3}}+(3^3)^{\frac{1}{3}}\right]\right]^{\frac{1}{4}} = \left[5\left(2^{3\times\frac{1}{3}}+3^{3\times\frac{1}{3}}\right)\right]^{\frac{1}{4}}$$
$$= [5\,(2+3)]^{\frac{1}{4}} = (5^2)^{\frac{1}{4}} = 5^{2\times\frac{1}{4}} = 5^{\frac{1}{2}} = \sqrt{5}$$

21. Simplify $8^{\frac{2}{3}}-\sqrt{9}\times 10^0+\left(\frac{1}{144}\right)^{\frac{-1}{2}}$. **[CBSE 2011]**

Sol.
$$8^{\frac{2}{3}}-\sqrt{9}\times 10^0+\left(\frac{1}{144}\right)^{-\frac{1}{2}} = (2^3)^{\frac{2}{3}}-\sqrt{3^2}\times 1+\left(\frac{1}{12^2}\right)^{-\frac{1}{2}} \qquad [a^0=1]$$
$$= 2^{3\times\frac{2}{3}}-(3^2)^{\frac{1}{2}}+\frac{1}{12^{-2\times\frac{1}{2}}}$$
$$= 2^2-3+\frac{1}{12^{-1}} = 4-3+12$$
$$= 16-3 = 13$$

22. Simplify $\sqrt[4]{81x^8y^4z^{16}}$. **[CBSE 2014, 2015]**

Sol.
$$\sqrt[4]{81x^8y^4z^{16}} = (81x^8\,y^4z^{16})^{\frac{1}{4}} = (81)^{\frac{1}{4}}\times(x^8)\times(y^4)^{\frac{1}{4}}\times(z^{16})^{\frac{1}{4}}$$
$$= 3^{4\times\frac{1}{4}}\times x^{8\times\frac{1}{4}}\times y^{4\times\frac{1}{4}}\times z^{16\times\frac{1}{4}} \qquad [(a^m)^n = a^{mn}]$$
$$= 3\times x^2\times y\times z^4 = 3x^2y\,z^4$$

23. Simplify $\dfrac{9^{\frac{1}{3}}\times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}}\times 3^{-\frac{2}{3}}}$. **[CBSE 2016]**

Sol.
$$\frac{9^{\frac{1}{3}}\times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}}\times 3^{-\frac{2}{3}}} = \frac{(3^2)^{\frac{1}{3}}\times(3^3)^{-\frac{1}{2}}}{3^{\frac{1}{6}-\frac{2}{3}}} = \frac{3^{\frac{2}{3}}\times 3^{-\frac{3}{2}}}{3^{-\frac{3}{6}}} = \frac{3^{\frac{2}{3}-\frac{3}{2}}}{3^{-\frac{1}{2}}} = \frac{3^{-\frac{5}{6}}}{3^{-\frac{1}{2}}}$$
$$= 3^{-\frac{5}{6}}.3^{\frac{1}{2}} = 3^{\frac{1}{2}-\frac{5}{6}} = 3^{\frac{3-5}{6}} = 3^{-\frac{2}{6}} = 3^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{3}}$$

Long Answer Type Questions [4 Marks]

24. If x is a positive real number and the exponents are rational numbers, then simplify:

$$\left(\frac{x^b}{x^c}\right)^{b+c-a}\times\left(\frac{x^c}{x^a}\right)^{c+a-b}\times\left(\frac{x^a}{x^b}\right)^{a+b-c}$$ **[CBSE 2011, 2016]**

Sol. Given
$$\left(\frac{x^b}{x^c}\right)^{b+c-a}\times\left(\frac{x^c}{x^a}\right)^{c+a-b}\times\left(\frac{x^a}{x^b}\right)^{a+b-c} = \frac{(x^b)^{(b+c-a)}}{(x^b)^{a+b-c}}\times\frac{(x^c)^{c+a-b}}{(x^c)^{b+c-a}}\times\frac{(x^a)^{a+b-c}}{(x^a)^{c+a-b}}$$
$$= \frac{(x)^{b^2+bc-ab}}{(x)^{ab+b^2-bc}}\times\frac{(x)^{c^2+ca-bc}}{(x)^{bc+c^2-ac}}\times\frac{(x)^{a^2+ab-ac}}{(x)^{ac+a^2-ab}}$$

$$= \frac{(x)^{b^2+bc-ab+c^2+ac-bc+a^2+ab-ac}}{(x)^{ab+b^2-bc+bc+c^2-ac+ac+a^2-ab}} \qquad [\because x^m \times x^n \times x^p = x^{m+n+p}]$$

$$= \frac{(x)^{a^2+b^2+c^2}}{(x)^{a^2+b^2+c^2}}$$

$$= (x)^{(a^2+b^2+c^2)-(a^2+b^2+c^2)} \qquad \left[\frac{x^m}{x^n} = x^{m-n}\right]$$

$$= x^0$$

$$= 1 \qquad [x^0 = 1]$$

25. **Simplify** $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{4}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$. **[CBSE 2016]**

Sol.

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{4}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right] = \left[\left(\frac{3}{2}\right)^4\right]^{-\frac{3}{4}} \times \left\{\left[\left(\frac{5}{2}\right)^2\right]^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right\}$$

$$= \left(\frac{3}{2}\right)^{-4\times\frac{3}{4}} \times \left[\left(\frac{5}{2}\right)^{-2\times\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right] = \left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{2}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{2}{3}\right)^3 \times \left[\left(\frac{5}{2}\right)^{-3+3}\right] \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$$

$$= \frac{2^3}{3^3} \times \left(\frac{5}{2}\right)^0$$

$$= \frac{8}{27} \times 1 \qquad [a^0 = 1]$$

$$= \frac{8}{27}$$

PRACTICE QUESTIONS BASED ON EXERCISE 1.6

1. Simplify: (*i*) $15^2 . 15^4$ (*ii*) $\frac{27^{10}}{27^6}$
2. Simplify: (*i*) $11^{\frac{2}{3}} . 11^{\frac{1}{3}}$ (*ii*) $13^{\frac{1}{5}} . 17^{\frac{1}{5}}$
3. Find: (*i*) $15^2 . 15^{-5}$ (*ii*) $(17^2)^{-1}$
4. Evaluate: (*i*) $(2^2)^3$ (*ii*) $\left(\frac{1}{3^5}\right)^4$
5. Write the simplified form of $\frac{13^{\frac{1}{5}}}{13^{\frac{1}{3}}}$. **[CBSE 2011]**
6. Simplify: (*i*) $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$ (*ii*) $\left(\frac{15^{\frac{1}{4}}}{3^{\frac{1}{2}}}\right)$ (*iii*) $\left(\frac{12^{\frac{1}{5}}}{27^{\frac{1}{5}}}\right)$ **[CBSE 2011]**
7. Find: (*i*) $\sqrt[4]{\sqrt[3]{2^2}}$ (*ii*) $2\sqrt[5]{\sqrt[4]{(2^3)^4}}$
8. Find the value of

   (*i*) $\sqrt{(144)^{-2}}$ (*ii*) $\sqrt{(3)^{-2}}$
9. Simplify $\sqrt[3]{2} \times \sqrt[4]{3}$. **[HOTS]**
10. It is given that m and n are two natural numbers such that $m^n = 32$. Find the value of n^{mn}. **[HOTS]**
11. Find the value of x for which $\left(\frac{3}{4}\right)^6 \times \left(\frac{16}{9}\right)^5 = \left(\frac{4}{3}\right)^{x+2}$.
12. Evaluate: (*i*) $(3^2 + 4^2)^{\frac{1}{2}}$ (*ii*) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$
13. If $2^x \times 4^x = 8^{\frac{1}{3}} \times (32)^{\frac{1}{5}}$, then find the value of x. **[CBSE 2013]**

14. Write the value of $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$.

15. Simplify $\left\{5\left(16^{\frac{1}{4}}+27^{\frac{1}{3}}\right)\right\}^{\frac{1}{4}}$.

16. Prove that $9^{\frac{3}{2}} - 3 \times 2^{\circ} - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$.

17. Find the value of x, if $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$.

18. Evaluate $(27)^{-\frac{1}{3}}.(27)^{-\frac{1}{3}} \times \left[27^{\frac{1}{3}} - 27^{\frac{2}{3}}\right]$.

19. Prove that $\sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1} \times 5} \times \sqrt[6]{3 \times 5^5} = \frac{3}{5}$.

20. Simplify and give the result in exponent form:

$$\frac{(25)^{\frac{5}{2}} \times (729)^{\frac{1}{2}}}{(125)^{\frac{2}{3}} \times (27)^{\frac{2}{3}} \times (8)^{\frac{4}{3}}}.$$ **[HOTS]**

Value Based Questions

1. Two classmates Salma and Anil simplified two different expressions during the revision hour and explained to each other their simplifications. Salma explains simplification of $\frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ and Anil explains simplification of $\sqrt{28}+\sqrt{98}+\sqrt{147}$.

   Write both the simplifications. What values does it depict? **[CBSE 2013]**

2. Two friends Avvye and Shubham were planting trees along the boundary of their society. While doing this, Avvye said that they could only plant the number of trees which would be a natural number or a positive integer. Shubham agreed with his thought.
   (*i*) Are both of them correct?
   (*ii*) Define natural number, integers, rational and irrational number.
   (*iii*) What values are depicted from their activity?

3. For a rally against corruption, students were having boards written "SAY NO TO CORRUPTION". The boards are square-shaped of each side 3 m. One of the student states that the diagonal of the board is an irrational number.
   (*i*) Do you agree with his statement and why?
   (*ii*) Locate this irrational number on the number line.
   (*iii*) What values are depicted from their activity?

4. Two friends Pankaj and Siddharth went to an antique store to purchase 3^4 old coins of single type. They finally short-listed coins of two types A and B and decided to choose the one having lowest price. On being asked the price, the shopkeeper told them coin 'A' is priced at ₹ 2^7 each and coin B is priced at ₹ 3^5 each. Pankaj helped Siddharth to select the type of coin on the basis of price. They further decided to distribute coins among their 3^2 friends.
   (*i*) Which type do you think they finally bought based on lowest price? What is the total price they paid to the shopkeeper?
   (*ii*) Find the number of coins each of which their friends received.
   (*iii*) What values did Pankaj depict when he helped Siddharth?

INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

1. Is – 25 a rational number? Give reasons.
2. Insert a rational and an irrational number between 0.0001 and 0.001.
3. Classify the following numbers as rational or irrational with justification:
   (*i*) $-\sqrt{0.4}$ (*ii*) $(1+\sqrt{5})-(4+\sqrt{5})$
   [NCERT Exemplar]
4. Which is greater: $\sqrt[3]{16}$ or $\sqrt[5]{8}$?
5. Write in exponent form: $\sqrt[4]{\sqrt[3]{3^2}}$.
6. Find the value of $(256)^{0.16} \times (256)^{0.09}$. **[NCERT Exemplar]**
7. What should be multiplied and divided to rationalise the denominator of $\frac{1}{\sqrt{7}}$?
8. What is the value of $\sqrt[4]{(81)^{-2}}$?

Short Answer Type Questions I [2 Marks]

9. Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$.
10. Express 0.404040... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **[NCERT Exemplar]**

11. Simplify $12\sqrt{18} - 6\sqrt{20} - 3\sqrt{50} + 8\sqrt{45}$.

12. Show that $0.142857142857 \ldots = \frac{1}{7}$.

[NCERT Exemplar]

13. Simplify $\frac{5}{\sqrt{8}} + \frac{1}{\sqrt{3}}$.

14. Simplify $\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}} \ldots$. **[HOTS]**

15. Simplify $[1^3 + 2^3 + 3^3 + 4^3 + 5^3]^{\frac{1}{2}}$.

16. Rationalise the denominator and evaluate $\frac{\sqrt{2}}{2+\sqrt{2}}$ by taking $\sqrt{2} = 1.414$ up to three decimal places.

17. Evaluate $\sqrt[4]{12} \times \sqrt[7]{6}$. **[NCERT Exemplar]**

18. State whether the following statements are true or false. Give reasons for your answers.
    - (*i*) Number of rational numbers between 15 and 18 are finite.
    - (*ii*) There are numbers which cannot be written in the form $\frac{p}{q}$, where p and q both are integers and $q \neq 0$.
    - (*iii*) Decimal representation of rational numbers cannot be non-terminating non-repeating.
    - (*iv*) The product of any two irrational numbers is always irrational.

Short Answer Type Questions II [3 Marks]

19. Find which of the variable x, y, z and u represent rational numbers and irrational numbers:

    (*i*) $x^2 = 5$ (*ii*) $y^2 = 9$

    (*iii*) $z^2 = .04$ (*iv*) $u^2 = \frac{17}{4}$. **[NCERT Exemplar]**

20. Represent geometrically $\sqrt{8.1}$ on the number line.

[NCERT Exemplar]

21. Simplify $0.\overline{134} - 0.00\overline{32}$ and express the result in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

22. Find the value of a and b,

    if $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}\sqrt{5}\,b$. **[NCERT Exemplar]**

23. Find the value of x, if $5^{x-3} \times 3^{2x-8} = 225$. **[HOTS]**

24. Simplify $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{80}+\sqrt{48}-\sqrt{27}-\sqrt{45}}$.

25. Assuming that x and y are positive real numbers, simplify the following:

    (*i*) $\left(x^{\frac{-2}{3}} \times y^{-\frac{1}{2}}\right)^2$

    (*ii*) $(\sqrt{x})^{-\frac{2}{3}} . \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$.

26. Represent the real number $\sqrt{7}$ on the number line.

27. Visualise the representation of $5.3\overline{7}$ using successive magnification up to 4 decimal places, that is up to 5.3777.

Long Answer Type Questions [4 Marks]

28. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **[NCERT Exemplar]**

29. Simplify $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$.

[NCERT Exemplar]

30. If $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$, then find the value of $\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$. **[NCERT Exemplar]**

31. Prove that $\frac{2^{36} + \left(\frac{1}{4} \times 2^{35}\right) + \left(\frac{1}{8} \times 2^{37}\right)}{\left(\frac{1}{16} \times 2^{39}\right) + \left(\frac{1}{8} \times 2^{38}\right)} = \frac{11}{8}$. **[HOTS]**

32. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find $x^4 + \frac{1}{x^4}$.

33. Evaluate

    $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{9}+\sqrt{8}}$.

[HOTS]

34. If $x = 9 + 4\sqrt{5}$, find the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$.

35. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + xy - y^2$.

ASSESS YOURSELF

1. Is 1.010010001 ...... an irrational number? If so, why?
2. Give examples of two irrational numbers, the product of which is
   (*i*) a rational number (*ii*) an irrational number
3. Simplify $(5+\sqrt{2})(3+\sqrt{5})$.
4. Simplify $(\sqrt[3]{x^2})^{\frac{3}{2}}$.
5. Insert two rational numbers between $\frac{2}{3}$ and $\frac{5}{3}$.
6. Simplify $64^{-\frac{1}{3}} . 64^{\frac{1}{3}} - 343^{\frac{2}{3}}$.
7. Write two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.
8. Find the value of x, if $\sqrt[3]{4x-7} = 5$.
9. Let x and y be rational and irrational numbers, respectively. Is $x + y$ necessarily an irrational number? Give an example in support of your answer.
10. Locate $\sqrt{3}$ on the number line. **[CBSE 2016]**
11. Express 2.5434343 .... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
12. Simplify $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$.
13. Evaluate $4 \times (81)^{-\frac{1}{2}} \times \left[81^{\frac{1}{2}} + 81^{\frac{3}{2}}\right]$.
14. Simplify $\left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6 \div \left(\frac{3^4 \times 5^2}{2^6}\right)$.
15. Find the value of $\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{64}}$.
    [CBSE 201
16. Prove that $\left(\frac{x^l}{x^m}\right)^{\frac{1}{lm}} \times \left(\frac{x^m}{x^n}\right)^{\frac{1}{mn}} \times \left(\frac{x^n}{x^l}\right)^{\frac{1}{nl}} = 1$.
17. Locate $\sqrt{10}$ on the number line.
18. If $2^a = 3^b = 6^c$, then find the relation between a, and c.
19. Given $\sqrt{2} = 1.4142$ and $\sqrt{6} = 2.4495$. Find the valu of $\frac{1}{\sqrt{3}-\sqrt{2}-1}$ correct to three places of decima
    [CBSE 201
20. If $x = \frac{5-\sqrt{21}}{2}$, then prove that
    $$\left(x^3+\frac{1}{x^3}\right)-5\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)=0.$$
    [CBSE 201